

# Games, graphs, and machines



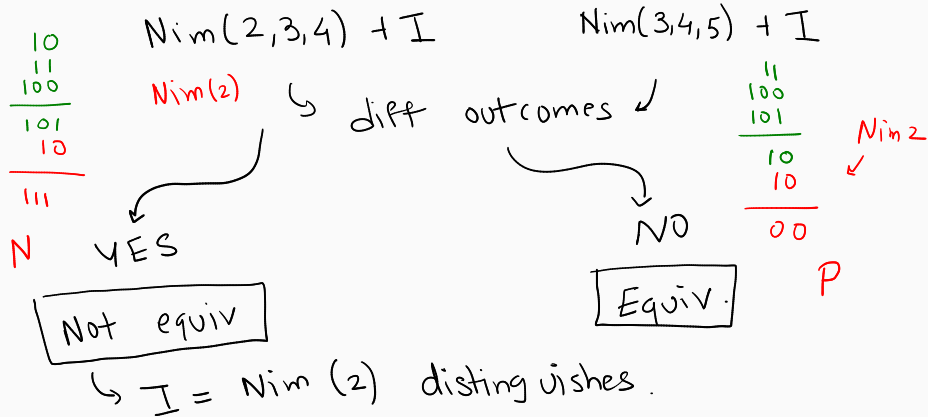
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# Equivalent?

Are Nim(2, 3, 4) and Nim(3, 4, 5) equivalent?

Can you find game I so that



Thm: Two nims of different nim-sum are inequivalent.

$$\begin{array}{lcl} \text{Nim}(\dots, \dots) & \rightarrow & a \\ \text{Nim}(\dots, \dots) & \rightarrow & b \end{array} \quad a \neq b$$

+ Nim(a) makes first P  
second N

+ Nim(h)      —||—      second P  
first N

# Equivalent?

Is there a Nim game that distinguishes  $\text{Nim}(2, 3, 4)$  and  $\text{Nim}(5)$ ?

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 $101$

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 $101$

# Equivalent?

Is there a Nim game that distinguishes  $\text{Nim}(2, 3, 4)$  and  $\text{Nim}(5)$ ?

Is there any game that distinguishes them? NO

$\text{Nim} \rightsquigarrow \text{nim-sum}$

Any game  $\rightsquigarrow$  "Grundy label".

$$G + P \sim G$$

Let H be a P game.

Then  $G \& G+H$  have same outcome.

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$G$  is N  $\Rightarrow G+H$  also N ✓

$G$  is P  $\Rightarrow G+H$  also P. ✓

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$$G+H \sim G$$



P-game.

means

$G+H+I \& G+I$   
same outcome.

# With apologies to Tolstoy

*All P-games are alike; each N-game is N in its own way.*

why?



$H_1$  P game

$H_2$  P game

Claim:  $H_1 \sim H_2$



$H_1 + I$

"

outcome (I)

$H_2 + I$

"

outcome (I)

$H$ -P game

$\sim$

All P-games

Nim games

Nim(1)

Nim(2)

Nim(3)

⋮

Preview : Two games are eqv. iff  
they have same "nim-sum".  
"Grundy label".